2.6

$$\left[\begin{array}{cccccc|c} 0&1&0&0&1&0&2\\ 0&0&0&1&1&0&-1\\ 0&1&0&0&0&1&1 \end{array}\right] \begin{array}{l} \\ \\ -R_1 \end{array}$$

$$\rightsquigarrow \left[\begin{array}{cccccc|c} 0&1&0&0&1&0&2\\ 0&0&0&1&1&0&-1\\ 0&0&0&0&1&-1&1 \end{array}\right]$$

$$\rightsquigarrow \left[\begin{array}{cccccc|c} 0&1&0&0&1&0&2\\ 0&0&0&1&1&0&-1\\ 0&0&0&0&1&-1&1 \end{array}\right] \begin{array}{l} -R_3 \\ -R_3 \\ \\ \end{array}$$

$$\rightsquigarrow \left[\begin{array}{cccccc|c} 0&1&0&0&0&1&1\\ 0&0&0&1&0&1&-2\\ 0&0&0&0&1&-1&1 \end{array}\right] \qquad \begin{array}{l} x_5 = 1\\ x_4 = -2\\ x_2 = 1 \end{array}$$

$$1\begin{bmatrix}1\\0\\0\end{bmatrix} + (-2)\begin{bmatrix}0\\1\\0\end{bmatrix} + 1\begin{bmatrix}1\\0\\1\end{bmatrix} = \begin{bmatrix}1\\-2\\1\end{bmatrix} \ ?$$

$$\begin{array}{c} x_1\\x_2\\x_3\\x_4\\x_5\\x_6 \end{array} = \begin{bmatrix}0\\1\\0\\-2\\1\\0\end{bmatrix} \text{ particular}$$

$$\left[\begin{array}{cccccc|c} -1&0&0&0&0&0&0\\ 0&1&0&0&0&1&1\\ 0&0&-1&0&0&0&0\\ 0&0&0&1&0&1&-2\\ 0&0&0&0&1&-1&1\\ 0&0&0&0&0&-1&0 \end{array}\right]$$

particular

General solution:

$$S = \left\{ \begin{bmatrix}0\\1\\0\\-2\\1\\0\end{bmatrix} + \alpha_1 \begin{bmatrix}-1\\0\\0\\0\\0\\0\end{bmatrix} + \alpha_2 \begin{bmatrix}0\\0\\-1\\0\\0\\0\end{bmatrix} + \alpha_3 \begin{bmatrix}0\\1\\0\\1\\-1\\-1\end{bmatrix} \right\}, \quad \alpha_{1,2,3} \in \mathbb{R}$$

$\boxed{2.12}$ p. 67 См. решения на 67 стр.

$U_1 \cap U_2 = X$:
$$\left(X = \alpha_{1,1}\begin{bmatrix}1\\1\\-3\\1\end{bmatrix} + \alpha_{1,2}\begin{bmatrix}2\\-1\\0\\-1\end{bmatrix}\right) \wedge$$
$$\wedge \left(\alpha_{1,1}\begin{bmatrix}1\\1\\-3\\1\end{bmatrix} + \alpha_{1,2}\begin{bmatrix}2\\-1\\0\\-1\end{bmatrix} = \alpha_{2,1}\begin{bmatrix}-1\\-2\\2\\1\end{bmatrix} + \alpha_{2,2}\begin{bmatrix}2\\-2\\0\\0\end{bmatrix}\right)$$

$$\begin{bmatrix}1 & 2 & +1 & -2\\ 1 & -1 & +2 & +2\\ -3 & 0 & -2 & 0\\ 1 & -1 & -1 & 0\end{bmatrix}\begin{matrix}\\ -R_1\\ +3R_2\\ -R_2\end{matrix} \rightsquigarrow \begin{matrix}1 & 2 & -1 & 2\\ 0 & -1 & -1 & -4\\ 0 & 3 & -4 & -6\\ 0 & -2 & 3 & 2\end{matrix}\begin{matrix}\\ \\ +3R_2\\ -2R_2\end{matrix} \rightsquigarrow \begin{matrix}1 & 2 & -1 & 2\\ 0 & -1 & -1 & -4\\ 0 & 0 & -1 & -18\\ 0 & 0 & 5 & 10\end{matrix}\begin{matrix}\\ \\ \\ +5R_3\end{matrix} \rightsquigarrow \begin{matrix}1 & 2 & -1 & 2\\ 0 & -1 & -1 & -4\\ 0 & 0 & -1 & -18\\ 0 & 0 & 0 & -80\end{matrix}$$

$\alpha_{1,1} = 0 \quad \alpha_{2,2} = 0$

$\alpha_{1,2} = 0 \quad \alpha_{2,1} = 0$?

$5\alpha_{2,1} = -10\alpha_{2,2}$

$\alpha_{2,1} = -2\alpha_{2,2}$

$-\alpha_{2,1} = 18\alpha_{2,2} \Rightarrow \alpha_{2,1} = -18\alpha_{2,2}$

$\alpha_{1,1}\left(-\frac{3}{2}\right) = \alpha_{2,1}$

$$\begin{bmatrix}-\frac{1}{2} & 2 & -2\\ -2 & -1 & 2\\ 0 & 0 & 0\\ \frac{5}{2} & -1 & 0\end{bmatrix} \rightsquigarrow \begin{matrix}\frac{9}{2} & -2\\ -\frac{9}{2} & 2\\ 0 & 0\\ 0 & 0\end{matrix} \rightsquigarrow \frac{9}{2}\alpha_{1,1} = -2\alpha_{2,2}$$

$\alpha_{1,1} \quad \alpha_{1,2} \quad \alpha_{2,2}$

$-\frac{5}{2}\alpha_{1,1} = \alpha_{1,2}$

$$\boxed{\alpha_{2,2} = -\frac{9}{4}\alpha_{1,1}}$$

Верно!

Ответ

$$\text{span}\begin{bmatrix}-4\\1\\2\\1\end{bmatrix}$$

$$X = \alpha_{2,1}\cdot\left(-\frac{2}{3}\right)\begin{bmatrix}1\\1\\-3\\1\end{bmatrix} + \left(\frac{5}{2}\right)\cdot\left(-\frac{2}{3}\right)\begin{bmatrix}2\\-1\\0\\-1\end{bmatrix} = \alpha_{2,1}\begin{matrix}-\frac{2}{3}-\frac{10}{3}\\ -\frac{2}{3}+\frac{5}{3}\\ 2+0\\ -\frac{2}{3}+\frac{5}{3}\end{matrix} = \alpha_{2,1}\begin{bmatrix}-4\\1\\2\\1\end{bmatrix}$$

$\boxed{2.13}$

a) $\dim(U_1)$ - ?

$$U_1 = \left\{ x : \begin{bmatrix} 1 & 0 & 1 \\ 1 & -2 & -1 \\ 2 & 1 & 3 \\ 1 & 0 & 1 \end{bmatrix} x = 0 \right\}$$

$$\begin{matrix} 1 & 0 & 1 \\ 0 & -2 & -2 \\ 0 & 1 & 1 \end{matrix} \rightsquigarrow \begin{matrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{matrix} \Rightarrow \dim(U_1) = 2$$

$\dim(U_2)$ - ?

$$\begin{matrix} 3 & -3 & 0 \\ 1 & 2 & 3 \\ 7 & -5 & 2 \\ 3 & -1 & 2 \end{matrix} \begin{matrix} \frac{1}{3} \\ -\frac{1}{3}R_1 \\ -7R_2 \\ -R_1 \end{matrix} \rightsquigarrow \begin{matrix} 1 & -1 & 0 \\ 0 & 3 & 3 \\ 0 & -15 & -10 \\ \cancel{0} & \cancel{2} & \cancel{2} \end{matrix} \; \tfrac{1}{3} \rightsquigarrow \begin{matrix} 1 & -1 & 0 \\ 0 & 1 & 1 \end{matrix}$$

$$\Downarrow$$

$$\dim(U_2) = 2$$

b) bases of U_1, U_2 ?

$$U_2 = \text{span}\left(\begin{bmatrix} 3 & -3 \\ 1 & 2 \\ 7 & -5 \\ 3 & -1 \end{bmatrix}\right) \qquad U_1 = \text{span}\left(\begin{bmatrix} 1 & 0 \\ 1 & -2 \\ 2 & 1 \\ 1 & 0 \end{bmatrix}\right)$$

c) basis of $U_1 \cap U_2$

$$\alpha_{1,1}\begin{bmatrix} 3 \\ 1 \\ 7 \\ 3 \end{bmatrix} + \alpha_{1,2}\begin{bmatrix} -3 \\ 2 \\ -5 \\ -1 \end{bmatrix} = \alpha_{2,1}\begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix} + \alpha_{2,2}\begin{bmatrix} 0 \\ -2 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{matrix} 3 & -3 & -1 \\ 3 & -1 & -1 \end{matrix} \rightsquigarrow \begin{matrix} 3 & -3 & -1 \\ 0 & 2 & 0 \end{matrix} \qquad \frac{3\alpha_{1,1} = \alpha_{2,1}}{\to \alpha_{1,2} = 0}$$

$$\begin{array}{ccc} 0 & -3 & 0 \\ -2 & 2 & 2 \\ 1 & -5 & -1 \\ 0 & -1 & 0 \\ \alpha_{1,1} & \alpha_{1,2} & \alpha_{2,2} \end{array} \Rightarrow \alpha_{1,1} = \alpha$$