$\boxed{3.1}$ $\langle x, y\rangle := x_1 y_1 - (x_1 y_2 + x_2 y_1) + 2(x_2 y_2)$

$x, y \in \mathbb{R}^2$

$\langle x, y\rangle$ - Inner product?

- definite and positive: $\forall x$, $\langle x, x\rangle = 0 \Rightarrow x = 0$
- symm. bilinear mapping, $\exists B: V \times V \to \mathbb{R}$

коммут.?

$\langle x, y\rangle = \langle y, x\rangle$

same as linear but for two args;

— Pos. defin.?

$\langle x, x\rangle = x_1^2 - 2x_1x_2 + 2x_2^2 = 0$ M, J, при $x_1, x_2 \neq 0$?

$x_1(x_1 - x_2) + x_2(2x_2 - x_1) = 0$

$x_1 - x_2 \neq 0$, $x_1 \neq x_2$?

$x_1 - x_2\left(\frac{-x_2}{x_1 - x_2} + 1\right) = 0$

$\frac{x_1^2}{x_2^2} - 2\frac{x_1}{x_2} + 2 = 0$ нет в R

$y^2 - 2y + 2 = 0$ $\frac{2 \pm \sqrt{4-8}}{2}$

Pos. defin.

- Sym. bilinear?

$x_1y_1 - (x_1y_2 + x_2y_1) + 2(x_2y_2) = y_1x_1 - (y_1x_2 + y_2x_1) + 2(y_2x_2)$

bilinear? $\langle \alpha x + \beta x', y\rangle = \alpha\langle x, y\rangle + \beta\langle x', y\rangle$

$(\alpha x_1 + \alpha x_1')y_1 - (\alpha x_1 + \alpha x_1')\cdot y_2 + (\alpha x_2 + x_2')$

Yes

3.1 continued

$\langle \alpha x+\beta x', y\rangle = (\alpha x_1+\beta x_1')y_1 - ((\alpha x_1+\beta x_1')y_2 + (\alpha x_2+\beta x_2')y_1) + 2((\alpha x_2+\beta x_2')y_2)$

$= \alpha(x_1y_1 - (x_1y_2 + x_2y_1) + 2x_2y_2) + \beta(\dots) = \alpha\langle x,y\rangle + \beta\langle x',y\rangle$

$\langle x, \alpha y+\beta y'\rangle = \dots = \alpha\langle x,y\rangle + \beta\langle x,y'\rangle$

$\square$

Проверил.

3.2 $\langle x,y\rangle = x^T\begin{bmatrix}2 & 0\\ 1 & 2\end{bmatrix} y$, $x,y \in R^2$

Inner product?
- Positive, symmetric, definite.
- Bilinear

$[x_1\ x_2]\begin{bmatrix}2 & 0\\ 1 & 2\end{bmatrix}\begin{bmatrix}y_1\\ y_2\end{bmatrix} = [x_1\ x_2]\begin{bmatrix}2y_1\\ y_1+2y_2\end{bmatrix} =$

$= \boxed{2x_1y_1 + y_1x_2 + 2x_2y_2}$

$\begin{array}{cccc} x_1 & x_2 & y_1 & y_2\\ - & - & + & +\end{array}$ ← Not positive? Не верно! Для $\langle x,x\rangle$

$y_1\left(2x_1 + \frac{x_2}{2}\right) + x_2\left(\frac{y_1}{2} + 2y_2\right) = 0,$

$\frac{y_1}{\cancel{2}} + \frac{x_2}{4x_1+x_2}\cdot\left(\frac{y_1+4y_2}{\cancel{2}}\right) = 0,\quad 4x_1+x_2 \neq 0$

$\frac{x_2}{4x_1+x_2} = -\frac{y_1}{y_1+4y_2},\quad 4x_1+x_2\neq 0,\quad y_1+4y_2\neq 0,\quad x,y\neq 0$

$x^T\begin{bmatrix}2 & 0\\ 1 & 2\end{bmatrix} y = 0$

$\frac{1}{4\underbrace{\frac{x_1}{x_2}}_{=x'}+1} + \frac{1}{1+4\underbrace{\frac{y_2}{y_1}}_{=y'}} = 0 \Rightarrow \frac{1}{4x'+1} + \frac{1}{4y'+1} = 0$

$+2$

$4y'\cancel{+1} + 4x'+\cancel{1} = 0,\quad 4x'\neq -1,\quad 4y'\neq -1$

$y' = \frac{-2x'-1}{2} = -x' - \frac{1}{2} \Rightarrow \frac{y_2}{y_1} = -\frac{x_1}{x_2} - \frac{1}{2}$

Может ли $\langle x,x\rangle = 0$, при $x\neq 0$, $y\neq 0$?

Bilinear? 1) $\langle x+y, z\rangle = \langle x,z\rangle + \langle y,z\rangle$
2) $\langle \alpha x, y\rangle = \alpha\langle x,y\rangle$

$\langle x,x\rangle = 2x_1^2 + x_1x_2 + 2x_2^2$

$\begin{array}{cc} x_1 & x_2\\ - & -\\ - & +\end{array}$

$x_1\left(\frac{4x_1+x_2}{2}\right) + x_2\left(\frac{4x_2+x_1}{2}\right)$

$|x_1x_2| < \underbrace{2x_1^2 + 2x_2^2}_{\text{Всегда полож.}}$

$\langle x,x\rangle = 0 \Rightarrow x = 0$?

$\exists x\neq 0$ $2x_1^2 + x_1x_2 + 2x_2^2 = 0$

$2\frac{x_1^2}{x_2^2} + \frac{x_1}{x_2} + 2 = 0$

$2x'^2 + x' + 2 = 0$

$D = b^2 - 4ac = 1 - 16 < 0$

$\Downarrow$

Нет решения в R.

$x = 0 \Leftarrow$

Definite