$\boxed{3.1}$ $\langle x, y \rangle := x_1 y_1 - (x_1 y_2 + x_2 y_1) + 2(x_2 y_2)$

$x, y \in \mathbb{R}^2$

$\langle x, y \rangle$ - Inner product?

- definite and positive: $\forall x$, $\langle x,x \rangle = 0 \Rightarrow x = 0$
- symm. bilinear mapping, $\beta: V \times V \to \mathbb{R}$

коммут.?

$\langle x, y \rangle = \langle y, x \rangle$

same as linear but for two args;

- Pos. defin.?

$\langle x, x \rangle = x_1^2 - 2x_1 x_2 + 2x_2^2 = 0$ м.б. при $x_1, x_2 \neq 0$?

$x_1(x_1 - x_2) + x_2(2x_2 - x_1) = 0$

$x_1 - x_2 \neq 0$, $x_1 \neq x_2$?

$x_1 - x_2\left(\frac{-x_2}{x_1 - x_2} + 1\right) = 0$

нет в $\mathbb{R}$

$\frac{x_1^2}{x_2^2} - 2\frac{x_1}{x_2} + 2 = 0$

$y^2 - 2y + 2 = 0$ $\quad \frac{2 \pm \sqrt{4-8}}{2}$

$\Leftarrow$ Pos. defin.

- Sym. bilinear?

$x_1 y_1 - (x_1 y_2 + x_2 y_1) + 2(x_2 y_2) = y_1 x_1 - (y_1 x_2 + y_2 x_1) + 2(y_2 x_2)$

Yes

bilinear? $\langle \alpha x + \beta x', y \rangle = \alpha \langle x, y \rangle + \beta \langle x', y \rangle$

$(\alpha x_1 + \alpha x_1') y_1 - (\alpha x_1 + \alpha x_1') \cdot y_2 + (\alpha x_2 + x_2')$

3.1 continued

$\langle \alpha x+\beta x', y\rangle = (\alpha x_1+\beta x_1')y_1 - ((\alpha x_1+\beta x_1')y_2 + (\alpha x_2+\beta x_2')y_1) + 2((\alpha x_2+\beta x_2')y_2)$

$= \alpha(x_1y_1 - (x_1y_2 + x_2y_1) + 2x_2y_2) + \beta(\ldots) = \alpha\langle x,y\rangle + \beta\langle x',y\rangle$

$\langle x, \alpha y+\beta y'\rangle = \ldots = \alpha\langle x,y\rangle + \beta\langle x, y'\rangle$

$\square$

Проверил.

3.2 $\langle x,y\rangle = x^T\begin{bmatrix}2&0\\1&2\end{bmatrix}y$, Inner product?

$x,y\in\mathbb{R}^2$

- Positive, symmetric, definite.
- Bilinear

$[x_1\ x_2]\begin{bmatrix}2&0\\1&2\end{bmatrix}\begin{bmatrix}y_1\\y_2\end{bmatrix} = [x_1\ x_2]\begin{bmatrix}2y_1\\y_1+2y_2\end{bmatrix} =$

$= \boxed{2x_1y_1 + y_1x_2 + 2x_2y_2}$ $\quad x_1\ x_2\ y_1\ y_2$ ← Not positive ? Не верно! Для $\langle x,x\rangle$

$y_1(2x_1 + \frac{x_2}{2}) + x_2(\frac{y_1}{2} + 2y_2) = 0$,

$\frac{y_1}{\cancel{2}} + \frac{x_2}{4x_1+x_2}\cdot\left(\frac{y_1+4y_2}{\cancel{2}}\right) = 0$, $4x_1+x_2 \neq 0$

$\frac{x_2}{4x_1+x_2} = -\frac{y_1}{y_1+4y_2}$, $\quad x^T\begin{bmatrix}2&0\\1&2\end{bmatrix}y = 0$

$4x_1+x_2\neq 0$

$y_1+4y_2\neq 0$, $x,y\neq 0$

$\frac{1}{4\frac{x_1}{x_2}+1} + \frac{1}{1+4\frac{y_2}{y_1}} = 0$

$\frac{x_1}{x_2} = x'$, $\frac{y_2}{y_1} = y'$ $\Rightarrow \frac{1}{4x'+1} + \frac{1}{4y'+1} = 0$

$4y' + \cancel{1} + 4x' + \cancel{1} = 0$, $4x'\neq 1$, $4y'\neq -1$

$y' = \frac{-2x'-1}{2} = -x' - \frac{1}{2}$ $\Rightarrow \frac{y_2}{y_1} = -\frac{x_1}{x_2} - \frac{1}{2}$

Может ли $\langle x,x\rangle = 0$, при $x\neq 0$, $y\neq 0$?

$\langle x,x\rangle = 2x_1^2 + x_1x_2 + 2x_2^2$ $\quad x_1, x_2$

$x_1\left(\frac{4x_1+x_2}{2}\right) + x_2\left(\frac{4x_2+x_1}{2}\right)$

$|x_1x_2| < 2x_1^2 + 2x_2^2$

Всегда положит.

$\langle x,x\rangle = 0 \Rightarrow x = 0$?

$\exists x\neq 0$

$2x_1^2 + x_1x_2 + 2x_2^2 = 0$

$2\frac{x_1^2}{x_2^2} + \frac{x_1}{x_2} + 2 = 0$

$2x'^2 + x' + 2 = 0$

$D = b^2 - 4ac =$

$= 1 - 16 < 0$

Нет решения в $\mathbb{R}$.

$x = 0 \Leftarrow$ Definite

Bilinear ? 1) $\langle x+y, z\rangle = \langle x,z\rangle + \langle y,z\rangle$

2) $\langle \alpha x, y\rangle = \alpha\langle x,y\rangle$

$\underbrace{(x+y)^T\begin{bmatrix}2&0\\1&2\end{bmatrix}z}_{\mathbb{R}^2 = z'} =$ $z_1'(x_1+y_1) + z_2'(x_2+y_2) =$

$= z_1'x_1 + z_2'x_2 + z_1'y_1 + z_2'y_2 =$

$= x^Tz' + y^Tz' = \langle x,z\rangle + \langle y,z\rangle$

$\langle \alpha x, y\rangle = \alpha(x^Tz') = \alpha\langle x,z\rangle$ $\square$

Ответ: Да, Inner product

Не верно!

$\langle x,y\rangle = \langle y,x\rangle$? симметричность

$[1\ 0]\begin{bmatrix}2&0\\1&2\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} \stackrel{?}{=} [0\ 1]\begin{bmatrix}2&0\\1&2\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}$

$[1\ 0]\begin{bmatrix}0\\2\end{bmatrix} = 0 \quad [0\ 1]\begin{bmatrix}2\\1\end{bmatrix} = 1$

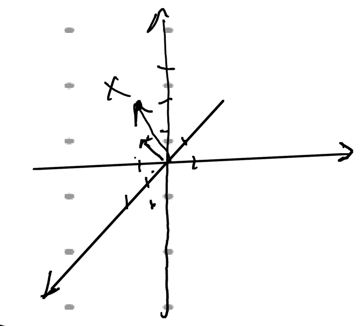
$0 \neq 1$!

Не симметрична

60m

$\boxed{3.3}$ 25m

$x = \begin{bmatrix}1\\2\\3\end{bmatrix}$ $y = \begin{bmatrix}-1\\-1\\0\end{bmatrix}$



a. $\langle x,y\rangle := x^Ty$

$x - y = z = \begin{bmatrix}2\\3\\3\end{bmatrix}$

$\|z\| = \sqrt{2^2+3^2+3^2} = \sqrt{22}$

Ошибка в solutions ?

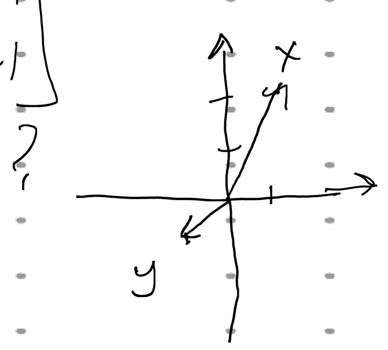
b. $\langle x,y\rangle := x^T\begin{bmatrix}2&1&0\\1&3&-1\\0&-1&2\end{bmatrix}y$

$\|z\| = \sqrt{[2\ 3\ 3]\begin{bmatrix}2&1&0\\1&3&-1\\0&-1&2\end{bmatrix}\begin{bmatrix}2\\3\\3\end{bmatrix}} = \sqrt{[2\ 3\ 3]\begin{bmatrix}7\\8\\3\end{bmatrix}} =$

$= \sqrt{14+24+9} =$

$= \sqrt{47}$

$\boxed{3.4}$ $x = \begin{bmatrix}1\\2\end{bmatrix}$, $y = \begin{bmatrix}-1\\-1\end{bmatrix}$



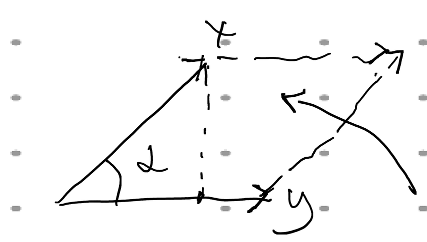
Angle b/w x and y?

Угол м/g x и y?

a) $\langle x,y\rangle = x^Ty$

$\omega = \arccos\frac{\langle x,y\rangle}{\|x\|\,\|y\|} =$

$= \frac{-1-2}{\sqrt{5\cdot 2}} = -\frac{3}{\sqrt{10}}$



x^Ty dot prod

$\cos\alpha = \frac{\langle x,y\rangle}{\|x\|\,\|y\|}$

≈ 2.82 rad

$\approx 161^0$ Верно

$\|x\| = \sqrt{\langle x,x\rangle}$

b) $\langle x,y\rangle = x^T\begin{bmatrix}2&1\\1&3\end{bmatrix}y$

$\omega = \arccos\frac{\begin{bmatrix}1\\2\end{bmatrix}^T\overbrace{\begin{bmatrix}2&1\\1&3\end{bmatrix}}^{B}\begin{bmatrix}-1\\-1\end{bmatrix}}{\sqrt{\underbrace{\left(\begin{bmatrix}1\\2\end{bmatrix}^T\begin{bmatrix}2&1\\1&3\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}\right)}_{4+14=18}\underbrace{\left(\begin{bmatrix}-1\\-1\end{bmatrix}^T\begin{bmatrix}2&1\\1&3\end{bmatrix}\begin{bmatrix}-1\\-1\end{bmatrix}\right)}_{3+4=7}}} =$ $\quad = (-4-7) = -11$

$= \arccos -\frac{11}{\sqrt{126}} \approx 168.5^0$

Ошибка в solutions.

Верно

$\boxed{3{,}5}$

a) $\Pi_U(x)$ — ?

$\Pi: X \to U$

$\underbrace{B^T B}\lambda = \underbrace{B^T x}$

$\downarrow$

λ

$\Pi_U(x) = B\lambda$

$$B = \begin{bmatrix} 0 & 1 & -3 & -1 \\ -1 & -3 & 4 & -3 \\ 2 & 1 & 1 & 5 \\ 0 & -1 & 2 & 0 \\ 2 & 2 & 1 & 7 \end{bmatrix}$$

← не базис.

базис! b_1, b_2, b_3

$$B^T B = \begin{bmatrix} 9 & 9 & 0 & 25 \\ 9 & 16 & -14 & 26 \\ 0 & -14 & 31 & 2 \\ 25 & 26 & 2 & 75 \end{bmatrix}$$

/ Неверно

$$B^T x = \begin{bmatrix} 9 \\ 23 \\ -25 \\ 31 \end{bmatrix} \qquad \lambda = \begin{bmatrix} -3 \\ 4 \\ 1 \\ -8{,}6 \end{bmatrix}$$

$$\lambda = \begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix}$$ / верно

$\Pi_U(x) = B\lambda = [1, -5, -1, -2, 3]^T$ / верно

$\boxed{3{,}5\,b}$

$$d(x, U) = \|\Pi_U(x) - x\| = \left\| \begin{bmatrix} 1 \\ -5 \\ -1 \\ -2 \\ 3 \end{bmatrix} - \begin{bmatrix} -1 \\ -9 \\ -1 \\ 4 \\ 1 \end{bmatrix} \right\| = \left\| \begin{bmatrix} 2 \\ 4 \\ 0 \\ -6 \\ 2 \end{bmatrix} \right\| =$$

$$= \sqrt{4 + 16 + 0 + 36 + 4} = \sqrt{60}$$

/ верно

$\boxed{3.6}$ $\langle x, y \rangle := x^T \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} y$; e_1, e_2, e_3 - stand basis

$\parallel$ A

a) $\pi_U(e_2) : e_2 \to U$ span$[e_1, e_3]$ $B = [e_1, e_3]$

$\overbrace{\langle \pi_U(e_2) - e_2}^{direct}, e_1 \rangle = 0$ I $\pi_U(e_2) = \frac{BB^T}{B^T B} e_2 =$ Неверно

$\langle \pi_U(e_2) - e_2, e_3 \rangle = 0$ II

$BB^T = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $B^T B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} e_1$

I) $= (\pi_U(e_2) - e_2)^T \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} = 0 \Rightarrow \pi_U(e_2)^T \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} = 1$

$(\lambda_1 e_1 + \lambda_2 e_3)^T \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} = 1 \to 2\lambda_1 = 1$

II) $= (\pi_U(e_2) - e_2)^T \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} = 0 \Rightarrow \left(\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}^T [e_1 + e_3] \right)^T \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} = -1 \to 2\lambda_2 = -1$

$$\boxed{\lambda_1 = \frac{1}{2} \quad \lambda_2 = -\frac{1}{2}}$$

$$\boxed{\pi_U(e_2) = \frac{e_1}{2} - \frac{e_3}{2}}$$

Верно

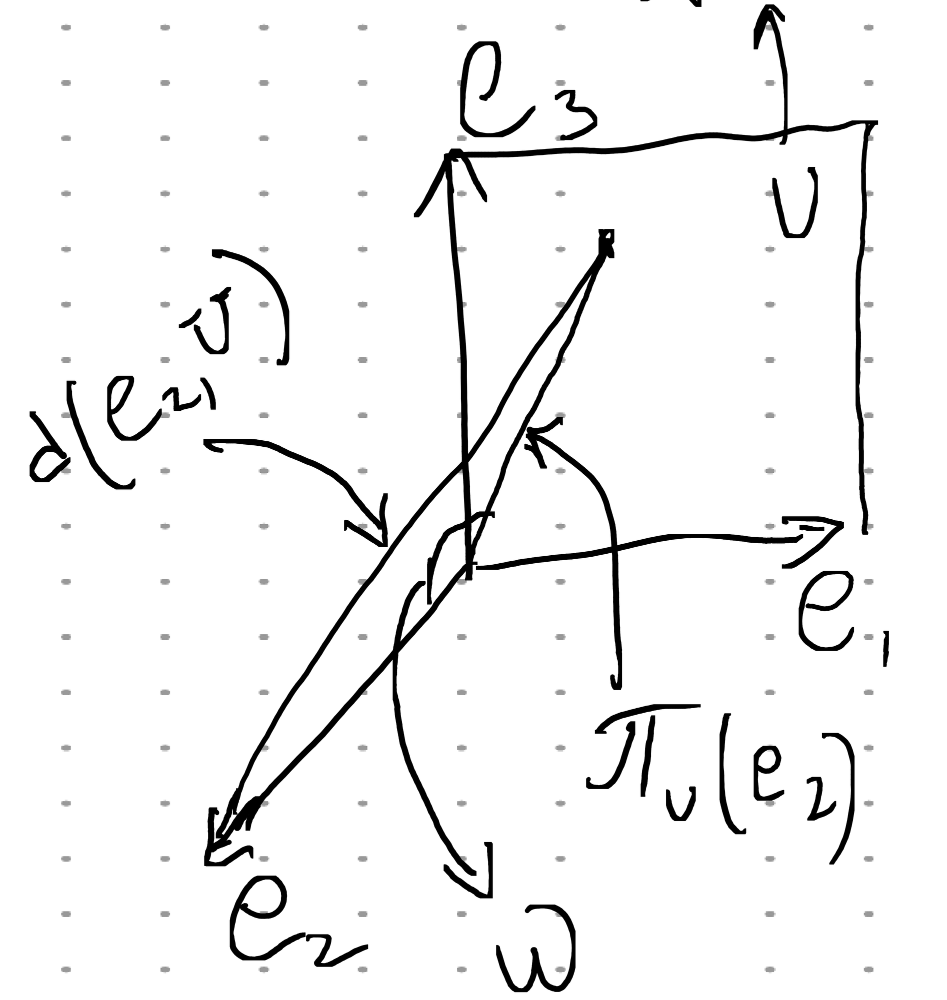
I + II $= \pi_U^T(e_2) \begin{bmatrix} 2 + 0 \\ 1 + [-1] \\ 0 + 2 \end{bmatrix} = 0$

$\left(\begin{bmatrix} \lambda_1 \\ 0 \\ \lambda_2 \end{bmatrix}^T [e_1 + e_3] \right)^T \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix} = 0$

$\parallel$ 1 0 1

$\underbrace{\lambda_1 e_1 + \lambda_2 e_3)}_{3\times 1} \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix} = 0$ $2\lambda_1 + 2\lambda_2 = 0$

$\begin{bmatrix} \lambda_1 \\ 0 \\ \lambda_2 \end{bmatrix}$



$\langle d(e_2, U), e_1 \rangle =$
$= \cos \omega \, |d| \, |e_1|$

3.6 b) Неверно

$d(e_2, U) = \langle \pi_U(e_2) - e_2, \pi_U(e_2) - e_2 \rangle =$

$\pi_U(e_2) - e_2 = \frac{1}{2} e_1 - \frac{e_3}{2} - e_2 = \begin{bmatrix} 1/2 \\ -1 \\ -1/2 \end{bmatrix}$

$d(e_2, U) = \begin{bmatrix} 1/2 \\ -1 \\ -1/2 \end{bmatrix}^T \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1/2 \\ -1 \\ -1/2 \end{bmatrix} =$

$= [0 \; 1 \; 0] \begin{bmatrix} 1/2 \\ -1 \\ -1/2 \end{bmatrix} = 1$

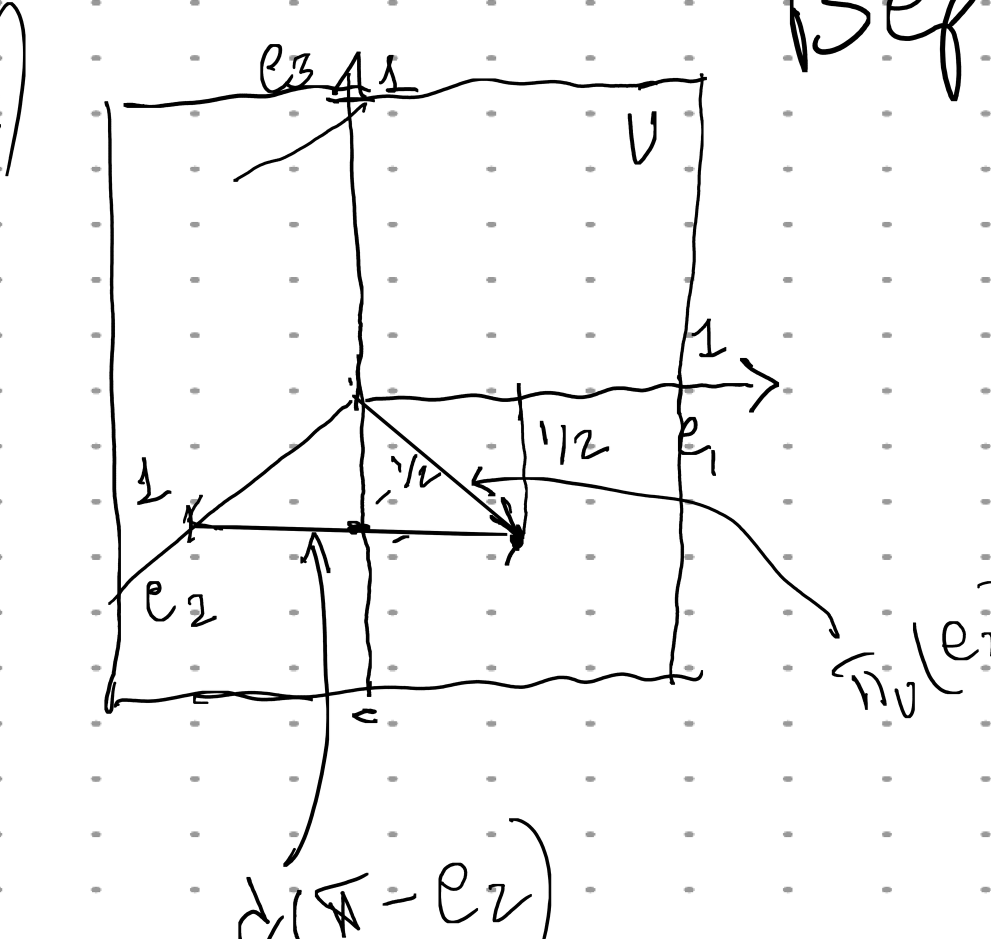
скалярн. произв.

$d(e_2, U) = \sqrt{\langle \pi - e_2, \pi - e_2 \rangle^2} =$

$= \sqrt{1^2} = 1$

Ответ. Верно

c)



$\boxed{3.7}$

V - vect. sp.

π - endormorp. ($\pi : V \to V$, linear)

a) π - proj. $\Leftrightarrow$ $(id_V - \pi)$ - proj.

id_V - $\underbrace{\text{id. endormor.}}$ on V (autom.?)

$id_V : V \to V$, $x \mapsto x$ biject.

π-proj: $\pi = \pi \circ \pi$

$\Rightarrow$) $\pi \circ \pi = \pi$

$(id_V - \pi) \circ (id_V - \pi) = \overbrace{id_V \circ id_V}^{id_V} - \overbrace{id_V \circ \pi}^{\pi} - \underbrace{\pi \circ id_V}_{\pi} + \underbrace{\pi \circ \pi}_{\pi} = (1)$

$id_V \circ id_V = id_V$

проекц. на себя

$id_V \circ (id_V - \pi) - \pi \circ (id_V - \pi)$

$(1) = id_V - 2\pi + \pi = id_V - \pi$ $\square$

$\Leftarrow$) $\exists$ $\underbrace{(id_V - \pi)^2}$ $= id_V - \pi$, $\pi \circ \pi = \pi$?

$\cancel{id_V} - 2\pi + \pi \circ \pi = \cancel{id_V} - \pi$

$\pi \circ \pi = \pi$ $\square$ ✓

$id_V - \pi : V \to W$

b) $\exists$ $\pi = \pi \circ \pi$

$Im(id_V - \pi)$ - ? for $im(\pi)$, $ker(\pi)$

$ker(id_V - \pi)$ - ?

$Im(id_V - \pi) \Rightarrow w \in W : (id_V - \pi)(v) = w$

π, id_V - linear $\Rightarrow$ $(id_V - \pi)$ - linear

$(id_V - \pi)(v) = id_V(v) - \underbrace{\pi(v)}_{im(\pi)} = v = im(\pi)$ $\square$

$ker(id_V - \pi)$

$ker(\Psi)$: $v \in V$: $\Psi(v) = 0$

$V \to W$

$(id_V - \pi)(v) = 0$

$id_V(v) - \pi(v) = 0$

$v - \pi(v) = 0$

$v = \pi(v)$

$ker(\pi)$: v, $\pi(v) = 0$

$ker(\pi) = v$: $\pi(v) = 0$

$ker(\pi) = v \Leftarrow \pi(v) = 0$?

b) $\exists x \in im(id_V - \pi)$

- $\pi \circ (id_V(x) - \pi(x)) = \pi(x) - \pi \circ \pi(x) = 0_V$

$\Downarrow$

$im(id_V - \pi) \subseteq ker\, \pi$

- $\exists x \in ker\, \pi$

$(id_V - \pi)(x) = id_V(x) = x$

$\Downarrow$

$ker\, \pi \subseteq im(id_V - \pi)$

$\boxed{ker\, \pi = im(id_V - \pi)}$

- $(id_V - \pi) \circ \pi = \pi - \pi = 0_V$

$\Downarrow$

$im\, \pi \subseteq ker(id_V - \pi)$

$\exists x = ker(id_V - \pi)$

$(id_V - \pi) = 0 \Rightarrow x - \pi(x) = 0 \Rightarrow x = \pi(x)$

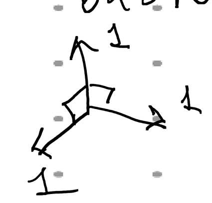
$ker(id_V - \pi) \in im(\pi)$

$\boxed{im\, \pi = ker(id_V - \pi)}$

$\square$

$\boxed{38}$ $B = \left(\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}-1\\2\\0\end{bmatrix}\right)$ for $U \subseteq \mathbb{R}^3$, $\cong \mathbb{R}^2$

Turn B to $C = (c_1, c_2)$ (ONB) ortho normal basis



$$c_1 = \frac{b_1}{\|b_1\|} = \frac{1}{\sqrt{3}}\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$c_2 = b_2 - \pi_{c_1}(b_2)$$

$$\tilde{\pi}_{c_1}(b_2) = \lambda c_1 = \frac{\overbrace{b_2^T c_1}\, c_1}{\underbrace{\|c_1\|^2}_{=1}} = (1)$$

$$\langle \pi_{c_1}(b_2) - b_2,\ c_1 \rangle = 0$$

$$\lambda\, c_1^T \cdot c_1 = b_2^T c_1$$

$$\lambda = \frac{b_2^T c_1}{\|c_1\|^2}$$

$$(1) = \underbrace{\left(-\frac{1}{\sqrt{3}} + \frac{2}{\sqrt{3}}\right)}_{\frac{1}{\sqrt{3}}} \cdot \frac{1}{\sqrt{3}}\begin{bmatrix}1\\1\\1\end{bmatrix} = \frac{1}{3}\begin{bmatrix}1\\1\\1\end{bmatrix} = \pi_{c_1}(b_2)$$

$$c_2' = b_2 - \pi_c(b_2) = \begin{bmatrix}-1-\frac{1}{3}\\ 2-\frac{1}{3}\\ -\frac{1}{3}\end{bmatrix} = \begin{bmatrix}-\frac{4}{3}\\ \frac{5}{3}\\ -\frac{1}{3}\end{bmatrix} = \frac{1}{3}\begin{bmatrix}-4\\5\\-1\end{bmatrix}$$

$$c_2 = \frac{c_2'}{\|c_2'\|} = \frac{c_2'}{\sqrt{\frac{16}{9}+\frac{25}{9}+\frac{1}{9}}} = \frac{1}{\sqrt{42}}\begin{bmatrix}-4\\5\\-1\end{bmatrix}$$

$$= \frac{\sqrt{42}}{3}$$

42

$$\|c_2\| = \sqrt{\frac{16}{42}+\frac{25}{42}+\frac{1}{42}} = \sqrt{1} = 1 \quad \square$$

Omnidna6 Solutions:

$$\frac{3}{\sqrt{42}}\begin{bmatrix}-4\\5\\-1\end{bmatrix}$$

[3.9]

$n\in N,\ \sum_{i=1}^{n} x_i = 1,\ x_i > 0,\ x_i \in R$ — e_1 norm

a) $\sum_{i=1}^{n} x_i^2 \ge \frac{1}{n}$

$x \cdot x = \|x\|^2$ dot prod

$\|x\| = \sqrt{\langle x, x\rangle}$ inner prod.

Коши-Буняковски: $|\langle x,y\rangle| \le \|x\|\cdot\|y\|$

$\Updownarrow$

$\langle x,y\rangle^2 \le \langle x,x\rangle\langle y,y\rangle$

Let $x_i = \frac{1}{n} \Rightarrow \sum x_i = n\frac{1}{n} = 1$

$v = [\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n}]^T$

$n \cdot \frac{1}{n^2} = \frac{1}{n}$

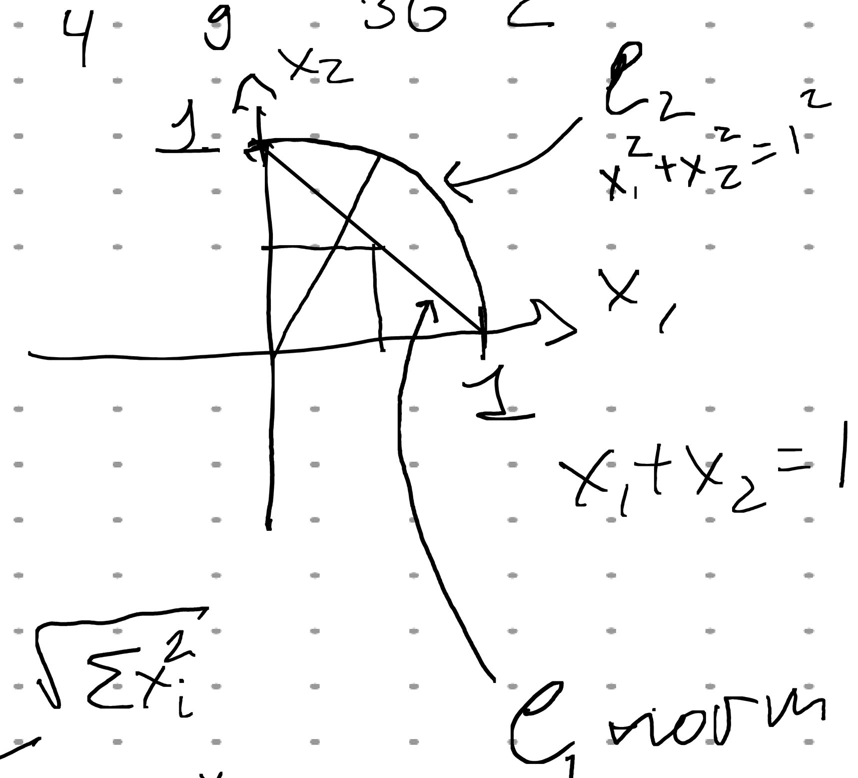
$\sum x_i^2 = \langle x,x\rangle \le \|x\|^2 = \left(\sqrt{\sum x_i^2}\right)^2$

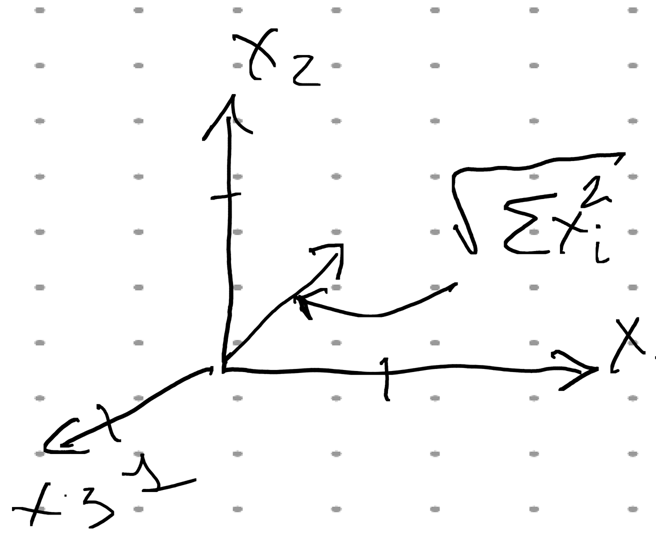
min $\langle x,x\rangle$ - ?

$x_i \ne x_j$

$[\frac{1}{2}, \frac{2}{3}]$

$\frac{1}{4} + \frac{4}{9} = \frac{25}{36} \ge \frac{1}{2}$





$\|x\| = \sqrt{\sum x^2}$

$\|x\|\cdot\|x\| = \sum x^2 \ge |\langle x,x\rangle| = \sum x_i^2$

$\frac{1}{a_1^2} + \frac{1}{a_2^2} + \dots + \frac{1}{a_n^2}$

$\sum \frac{1}{a_i} = \sum x_i = 1$

$\vec{x}, \vec{y}$

$|\langle x,y\rangle| \le \langle x,x\rangle\langle y,y\rangle$

$\sum x_i y_i \quad \sum^{n} x_i^2 \quad \sum y_i^2$

b) $\sum_{i=1}^{n} \frac{1}{x_i} \ge n^2$

$\sum x_i = 1,\ x_i > 0$

$|\langle x_i, y_i\rangle| \le \|x\|\,\|y\|$, $\sqrt{\langle y,y\rangle}$ $\Longrightarrow$ $\langle x,y\rangle^2 \le \langle x,x\rangle, \langle y,y\rangle$

$\sum_i \frac{\prod x_j}{\prod_k x_k} = \frac{\sum_i \prod_{i\ne j} x_j}{\prod_i x_i}$?

$\langle x,y\rangle^2 \le \langle x,x\rangle\langle y,y\rangle$

$\exists x [\{\frac{1}{\sqrt{x_i}}\}],\quad y = [\{\sqrt{x_i}\}]$

$\langle x,y\rangle = \sum \frac{\sqrt{x_i}}{\sqrt{x_i}} = n$

$\langle x,x\rangle = \sum \frac{1}{x_i} \quad \langle y,y\rangle = \sum x_i = 1$

$\Downarrow$

$n^2 \le \sum \frac{1}{x_i} \cdot 1$ ∎ Вярно

$\frac{\sum x_i y_i}{\sum y_i^2} \le \sum x_i^2,\ \sum y_i^2 > 0$

$\exists y_i = 1 \leftarrow$ But $\sum y_i = 1$? $y_i > 0$

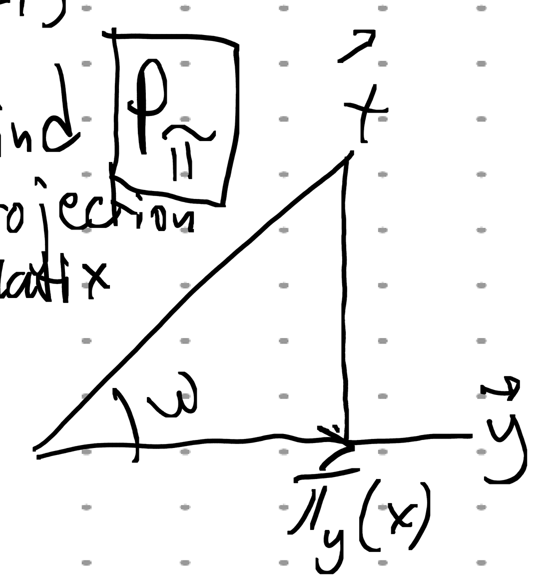
$\left(\sum x_i\right)^2 \le \left(\sum x_i^2\right)\cdot n$

$\frac{1}{n} \le \sum x_i^2$ Вярно

$\boxed{3.10}$

$x_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad x_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

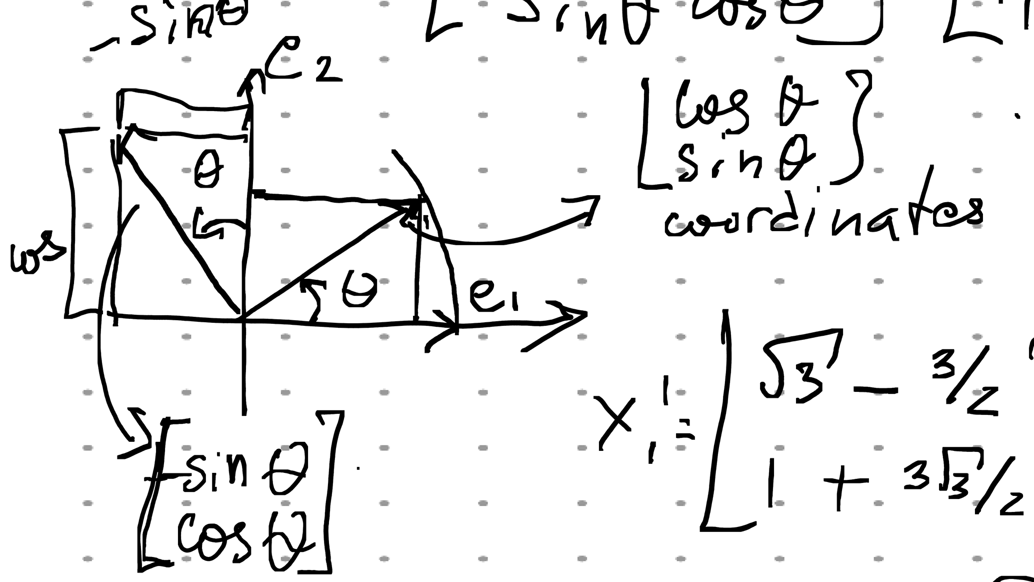
Rotate by 30° $=$ find $\boxed{P_{\pi}}$ projection Matrix



$|\cos w| = \dfrac{\|\pi_{x_1'}(x_1)\|}{\|x_1\|}$

$\cos w = \dfrac{\langle x, y\rangle}{\|x\| \underbrace{\|y\|}_{=1 \text{, normalized}}} \in \mathbb{R}$

$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \sqrt{3}/2 & -\frac{1}{2} \\ 1/2 & \sqrt{3}/2 \end{bmatrix}$



$x_1' = \begin{bmatrix} \sqrt{3} - 3/2 \\ 1 + 3\sqrt{3}/2 \end{bmatrix} = \begin{bmatrix} \frac{2\sqrt{3}-3}{2} \\ \frac{2+3\sqrt{3}}{2} \end{bmatrix} \approx \begin{bmatrix} 0{,}232 \\ 3{,}598 \end{bmatrix}$ ✓

$x_2' = \begin{bmatrix} 1/2 \\ -\sqrt{3}/2 \end{bmatrix} = \begin{bmatrix} 0{,}5 \\ -0{,}866 \end{bmatrix}$ ✓

v3 cf ttw