$\boxed{3.1}$ $\quad x, y \in R^2$

$\langle x,y\rangle = x_1y_1 - (x_1y_2 + x_2y_1) + 2(x_2y_2)$

– Inner product?

- symmetric? $\quad \langle x,y\rangle = \langle y,x\rangle$?

$y_1x_1 - (y_1x_2 + y_2x_1) + 2(y_2x_2) =$

$= x_1y_1 - (x_1y_2 + x_2y_1) + 2(x_2y_2)$ ✓

- positive definite?

$x \neq 0, \ \langle x,y\rangle > 0$? $\quad , \langle 0,0\rangle = 0$

$x_1y_1 - (x_1y_2 + x_2y_1) + 2(x_2y_2) = 0$

$x_1(y_1 - y_2) + x_2(-y_1 + 2y_2) = 0 \qquad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \neq 0$

$\exists\ x_1 = 0, x_2 \neq 0 \qquad\qquad\qquad\qquad\qquad \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \neq 0$

$y_1 = 0, y_2 \neq 0 \Rightarrow x_2 \cdot 2y_2 = 0 \Rightarrow x_2 \vee y_2 = 0$

противоречие

$\exists$

x_1	x_2	y_1	y_2	$x_1(y_1-y_2) + x_2(-y_1+2y_2)$
0	$\neq 0$	0	$\neq 0$	$x_2 \vee y_2 = 0$
0	$\neq 0$	$\neq 0$	0	$x_2 \cdot (-y_2) = 0$
$\neq 0$	0	0	$\neq 0$	$x_1 \cdot y_1 = 0$
$\neq 0$	0	$\neq 0$	0	$x_1 \cdot (-y_2) = 0$

противор.

$\Downarrow$

pos. def. $\Leftarrow \langle x,y\rangle \neq 0$

bilinear?

$\langle x, \lambda y + \psi z\rangle = \lambda\langle x,y\rangle + \psi\langle x,z\rangle$

$x_1(\lambda y_1 + \psi z_1) - \big(x_1(\lambda y_2 + \psi z_2) + x_2(\lambda y_1 + \psi z_1)\big)$

$+ 2x_2(\lambda y_2 + \psi z_2) =$

$= \lambda x_1y_1 - (\lambda x_1y_2 + \lambda x_2y_1) + 2\lambda x_2y_2$

$+ \psi x_1z_1 - (\psi x_1z_2 + \psi x_2z_1) + 2\psi x_2z_2 =$

$= \lambda\langle x,y\rangle + \psi\langle x,z\rangle \quad \square$

[illegible]