$\boxed{4.1}$

$$\begin{vmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 0 & 2 & 4 \end{vmatrix} = (-1)^2(16-12) + (-1)^3(8-0) + (-1)^4(4-0) =$$
$$= 4 - 8 + 4 = 0$$

Верно

$\boxed{4.2}$

$$\begin{array}{cc} \begin{bmatrix} 2 & 0 & 1 & 2 & 0 \\ 2 & -1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 1 & 2 \\ -2 & 0 & 2 & -1 & 2 \\ 2 & 0 & 0 & 1 & 1 \end{bmatrix} & \begin{array}{l} \\ -R_1 \\ +R_2 \\ +R_1 \\ -R_1 \end{array} \end{array} \rightsquigarrow \begin{bmatrix} 2 & 0 & 1 & 2 \\ 0 & -1 & -1 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & -1 & -1 \end{bmatrix}$$

$$\rightsquigarrow \begin{bmatrix} 2 & 0 & 1 & 2 & 0 \\ 0 & -1 & -1 & -1 & 1 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & -1 & 4 \end{bmatrix}$$

$\boxed{4,3}$ Eigenspace of

a) $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ $\quad Ax = \lambda x$

$(A - \lambda I)x = 0$

$\begin{bmatrix} 1-\lambda & 0 \\ 1 & 1-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \rightarrow |A - \lambda I_2| = (1-\lambda)^2 = 0$

$\Downarrow$

$\lambda = 1$

$\Downarrow$

$(A - 1I_2)x = 0$

$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

Ответ: span$\left\{\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right\}$

т.к. базис $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\begin{bmatrix} x_1(1-\lambda) \\ x_1 + x_2 - \lambda x_2 \end{bmatrix} = 0$

$x_1 = \lambda$

$x_2 - \lambda x_2 = -\lambda \quad x_2 = \frac{\lambda}{\lambda - 1}$

Char. polyn.:

$\det(A - \lambda I_2) = \underbrace{C_0}_{\det(A)} + \underbrace{C_1}_{(-1)^0 \cdot tr(A)}\lambda + C_2\lambda^{2=n} =$

$= 1 + 2\lambda - 2\lambda^2 = 0$

$\lambda_{1,2} = \frac{+4 \pm \sqrt{4+8}}{+4} = 1 \pm \sqrt{6}$

1) $x_1 = 1 + \sqrt{6}$, $x_2 = \frac{1+\sqrt{6}}{\sqrt{6}}$

2) $x_1 = 1 - \sqrt{6}$, $x_2 = \frac{\sqrt{6}-1}{\sqrt{6}}$

Неверно

b) $B = \begin{bmatrix} -2 & 2 \\ 2 & 1 \end{bmatrix}$

$|B - \lambda I| = \begin{vmatrix} -2-\lambda & 2 \\ 2 & 1-\lambda \end{vmatrix} = -2 + \cancel{\lambda} - \cancel{\lambda} + \lambda^2 - \ldots$ $+2\lambda + \lambda^2$

$= \lambda^2 - 6$

$= -2 - \lambda + 2\lambda + \lambda^2 - 4 = 0$

$\lambda^2 + \lambda - 6 = 0 \quad \lambda_{1,2} = \frac{-1 \pm \ldots}{\ldots}$

$= -1 \ldots$

$\lambda_1 = 2$

$\lambda_1 = 2!$

$(B - \lambda I)x = 0$

$\left[\begin{array}{cc|c} -4 & 2 & 0 \\ 2 & -1 & 0 \end{array}\right]$ /2 +

$\begin{bmatrix} -2 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \ldots$

$-2x_1 + x_2 = 0$

$\boxed{x = \text{span}\begin{bmatrix} 1 \\ 0 \end{bmatrix}}$ Неверно solut...

$\lambda_2 = -3$

$\begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix} \div 5 \quad \xrightarrow{-2R_1} \begin{bmatrix} 1 & \ldots \\ 0 & \ldots \end{bmatrix}$

4.4

$$A = \begin{bmatrix} 0 & -1 & 1 & 1 \\ -1 & 1 & -2 & 3 \\ 2 & -1 & 0 & 0 \\ 1 & -1 & 1 & 0 \end{bmatrix}$$

$$\det(A - \lambda I) = \underbrace{\det(A)}_{=2} + (-1)\underbrace{tr}_{=1}(A)\lambda + (-1)^2 tr(A)\lambda^2 + (-1)^3 tr\,\lambda^3 + (-1)^4\lambda^4$$

$$= 2 - \lambda + \lambda^2 - \lambda^3 + \lambda^4 = 2 - (1-\lambda)\cdot(\lambda + \lambda^3) = 0$$

$$2 - \lambda(1-\lambda) - \lambda^3(1-\lambda) =$$

$$\det(A - \lambda I) = (\lambda - 2)(\lambda - 1)(\lambda + 1)^2 = 0$$

$$\lambda_1 = 2,\ \lambda_2 = 1,\ \lambda_3 = -1$$

$(A - \lambda_i I$

$\lambda_1 = 2$:

$$\begin{bmatrix} -2 & -1 & 1 & 1 \\ -1 & -1 & -2 & 3 \\ 2 & -1 & -2 & 0 \\ 1 & -1 & 1 & -2 \end{bmatrix}$$

$$\begin{bmatrix} -2 & -1 & 1 & 1 \\ -1 & -1 & -2 & 3 \\ 2 & -1 & -2 & 0 \\ 1 & -1 & 1 & -2 \end{bmatrix} \begin{array}{l} \\ -R_1/2 \\ +R_1 \\ +R_2 \end{array} \sim \begin{array}{cccc} -2 & -1 & 1 & 1 \\ 0 & -1/2 & -5/2 & 5/2 \\ 0 & -2 & -1 & 1 \\ 0 & -2 & -1 & 1 \end{array} \begin{array}{l} /-2 \\ \times 2 \\ /-2\ +R_2 \\ -R_3 \end{array} \rightarrow \begin{array}{cccc} 1 & 1/2 & 1/2 & 1/2 \\ 0 & -1 & -5 & 5 \\ 0 & 0 & -5\frac{1}{2} & 4\frac{1}{2} \\ 0 & 0 & 0 & 0 \end{array}$$

basis

4.4) continued

$$\begin{array}{cl} \begin{bmatrix} -2 & -1 & 1 & 1 \\ -1 & -1 & -2 & 3 \\ 2 & -1 & -2 & 0 \\ 1 & -1 & 1 & -2 \end{bmatrix} & \begin{array}{l} \\ \\ +R_1 \\ + \end{array} \end{array}$$