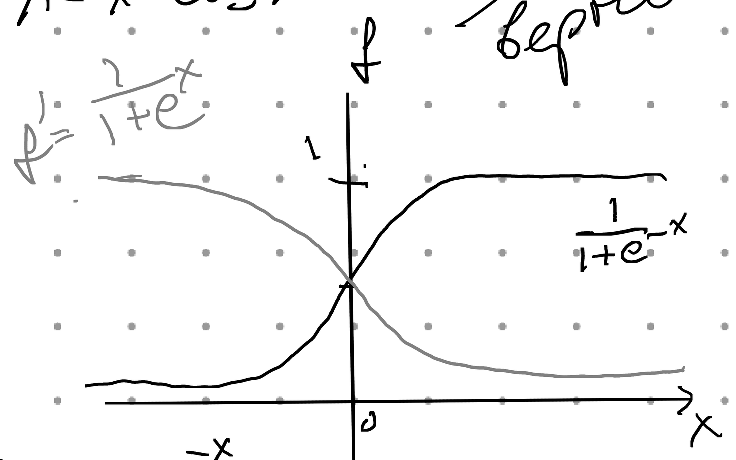
$\boxed{5.1}$ $f(x) = \log(x^4)\sin(x^3)$

$f'(x) = 4x^3 \cdot \frac{1}{x^4} \cdot \sin(x^3) + \log(x^4)\, 3x^2 \cos x^3 =$

$= \frac{4}{x}\sin(x^3) + \log(x)\, 12x^2 \cos x^3$

верно

$\boxed{5.2}$ $f(x) = \frac{1}{1+e^{-x}}$



$f' = -\frac{1}{(1+e^{-x})^2} \cdot e^{-x} \cdot (-1) = \frac{e^{-x}}{1+e^{-x}} = \frac{1}{1+e^{x}}$

Не верно

$f' = \frac{e^{-x}}{(1+e^{-x})^2} = \frac{1}{(e^x+1)(1+e^{-x})} =$

$= \left(\frac{e^x}{e^x} + e^x + \frac{1}{e^x} + 1\right)^{-1} = \frac{e^x}{e^{2x}+2e^x+1} = \frac{e^x}{(e^x+1)^2}$

верно

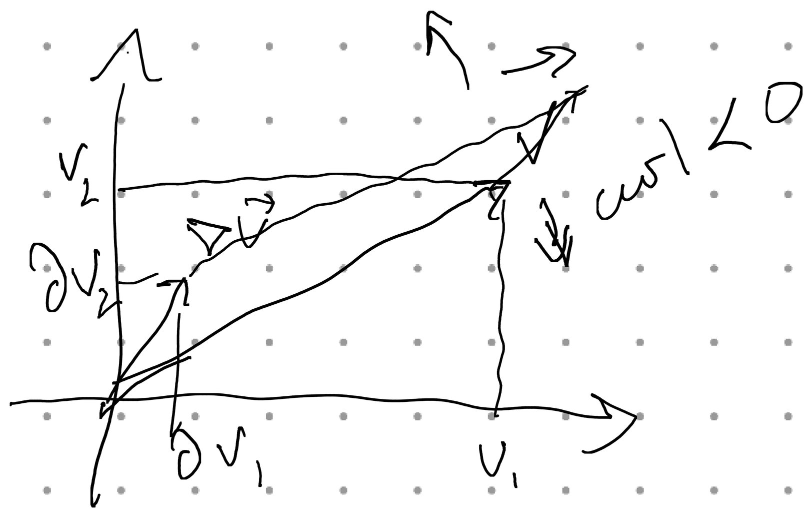
$\boxed{5.3}$ $f(x) = e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$, $\mu\,\sigma \in R$

$f' = \underbrace{e^{-\frac{1}{2\sigma^2}(x-\mu)^2}}_{f} \cdot -\frac{1}{\cancel{2}\sigma^2} \cdot \cancel{2}(x-\mu) = -\frac{x-\mu}{\sigma^2} f(x)$

верно

$\boxed{5.4}$ Taylor

curl 2d$(\vec{v})$



$$\begin{vmatrix} \frac{1}{\partial x} & \partial v_1 \\ \frac{1}{\partial y} & \partial v_2 \end{vmatrix} =$$

$$= \frac{\partial v_2}{\partial x} - \frac{\partial v_1}{\partial y} =$$

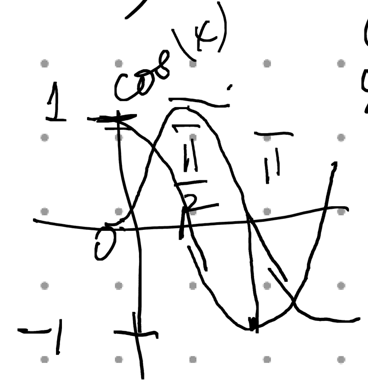
$$= ?$$

$\boxed{5.4}$ Taylor polynomials T_n

$f(x) = \sin(x) + \cos(x)$, $x_0 = 0$, $n = 0, \dots, 5$

$$T_0(x) = \frac{\overset{\sin^{(0)}(x) + \cos^{(0)}(x)}{\cancel{\sin(0) + \cos(0)}}}{0!}(x-0)^0 = (I)$$

$\cos'(x) = -\sin(x)$
$\sin'(x) = \cos(x)$



$$(I) = \frac{0 + (1)}{1}(x)^0 = 1^0 = T_0(x)$$

$$T_1(x) = \frac{\sin'(0) + \cos'(0)}{1!}(x-0)^1 = \frac{\overset{=1}{\cos(0)} - \overset{=0}{\sin(0)}}{1!}(x-1) = T_0 + x$$

$$T_2(x) = \frac{-\sin(0) - \cos(0)}{2!}(x)^2 = T_1 - \frac{x^2}{2}$$

$$T_3(x) = T_2 + \frac{-\cos(0) + \sin(0)}{3!}x^3 = T_2 - \frac{x^3}{3!}$$

$$T_4(x) = T_3 + \frac{\sin 0 + \cos(0)}{4!}x^4 = T_3 + \frac{x^4}{4!}$$

$$T_5(x) = T_4 + \frac{\cos 0 - \sin 0}{5!}x^5 = T_4 + \frac{x^5}{5!}$$

$$T_6(x) = T_5 + \frac{-\sin 0 - \cos 0}{6!}x^6 = T_5 - \frac{x^6}{6!}$$

Beftw

$\boxed{5.5}$

a) dim of $\frac{\partial f_i}{\partial \vec{x}}$

$$f_1(\underset{\in \mathbb{R}^2}{\vec{x}}) = \sin(x_1)\cos(x_2) \in \mathbb{R}$$

$$\frac{\partial f_1}{\partial \vec{x}} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} \\ \frac{\partial f_1}{\partial x_2} \end{bmatrix} \in \mathbb{R}^2 = \begin{bmatrix} \cos x_1 \cos x_2 \\ -\sin(x_1)\sin x_2 \end{bmatrix}$$

$$\frac{\partial f}{\partial x} = \frac{f(x + \partial x) - f(x)}{\partial x}$$

$$\underset{\in \mathbb{R}}{f_2(\vec{x}, \vec{y})} = \overbrace{\underset{1\times n}{\vec{x}^T} \underset{n}{\vec{y}}}^{1}, \quad \vec{x}, \vec{y} \in \mathbb{R}^n$$

$$\mathbb{R}^n \ni \frac{\partial f_2}{\partial \vec{x}} = \begin{bmatrix} \frac{\partial f_2(\vec{x}, \vec{y})}{\partial x_1} & \dots & \frac{\partial f_2(\vec{x}, \vec{y})}{\partial x_n} \end{bmatrix} = \begin{bmatrix} y_1 + x_2y_2 + \dots + x_ny_n \\ x_1y_1 + y_2 + \dots + x_ny_n \\ \vdots \\ x_1y_1 + \dots + y_n \end{bmatrix}^T$$

$$\mathbb{R}^{n\times n} \ni f_3(\vec{x}) = \overbrace{\underset{n}{\vec{x}} \cdot \underset{1\times n}{\vec{x}^T}}^{n\times n}, \quad \vec{x} \in \mathbb{R}^n$$

$$\frac{\partial f_3}{\partial \vec{x}} = \begin{bmatrix} \frac{\partial(x_1x_1)}{\partial x_1} & \dots & \frac{\partial(x_1x_n)}{\partial x_n} \\ \vdots & & \vdots \\ \frac{\partial(x_nx_1)}{\partial x_n} & \dots & \frac{\partial(x_nx_n)}{\partial x_n} \end{bmatrix} \in \mathbb{R}^{n\times n} = \begin{bmatrix} 2x_1 & x_2 & \dots & x_n \\ x_1 & 2x_2 & \dots & x_n \\ x_1 & x_2 & \dots & x_n \\ \vdots & & & \\ x_1 & x_2 & \dots & 2x_n \end{bmatrix}$$