6.4

H - из 1 сумки
¬H - из 2 сумки
E - манго

$P(H) = 0{,}4 = \frac{2}{5}$

$P(E|H) = \frac{4}{8} = \frac{1}{2}$

$P(\neg H) = 0{,}6 = \frac{3}{5}$

$P(E|\neg H) = \frac{4}{6} = \frac{2}{3}$

$P(E) = \underbrace{\frac{2}{5}\cdot\frac{1}{2}}_{1/5} + \underbrace{\frac{\cancel{3}}{5}\cdot\frac{2}{\cancel{3}}}_{2/5}$

$P(H|E) = \frac{P(H)\cdot P(E|H)}{P(E)} = \frac{1/5}{1/5+2/5} = \frac{1}{3} < \underbrace{\frac{2}{5}}_{P(H)}$

Верно

$\boxed{6.2}$

$$p\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = 0{,}4\, \mathcal{N}\left(\begin{bmatrix} 10 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) + 0{,}6\, \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 8{,}4 & 2{,}0 \\ 2{,}0 & 1{,}7 \end{bmatrix}\right)$$

a)

$p(x_1) = 0{,}4 \cdot \mathcal{N}(x_1 \mid 10, 1) + 0{,}6\, \mathcal{N}(x_1 \mid 0, 8{,}4)$ [illegible]

$p(x_2) = 0.4\, \mathcal{N}(2,1) + 0{,}6\, \mathcal{N}(0, 1.7)$ ✓

b)

$E(x_1) = 0.4 \cdot 10 + 0{,}6 \cdot 0 = 4$ ✓

$E(x_2) = 0.4 \cdot 2 + 0.6 \cdot 0 = 0.8$ ✓

Modes x_1: 10, 0

Modes x_2: 2, 0

Medians:

$\int_{-\infty}^{a} p(x)\, dx_1 = \frac{1}{2}$?

c) $E(x) = \int p(x)\, x\, dx =$

$= 0.4 \cdot \begin{bmatrix} 10 \\ 2 \end{bmatrix} + 0{,}6 \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 4/5 \end{bmatrix}$ Theorem 6.12

$0{,}4 + 0{,}6 = 1$

Mode:

$\frac{d\, p(x_1)}{dx_1} =$?

$\boxed{6.3}$ $p(x|\mu) = \mu^{x}(1-\mu)^{1-x}$, $x \in \{0, 1\}$

not / compiles

Beta conj. prior

$$\underbrace{p(\mu|x)}_{\text{posterior}} = \frac{p(x|\mu)\cdot \overbrace{p(\mu)}^{\text{prior}}}{p(x)}$$

Beta prior: $p(\mu|\alpha,\beta) \propto \mu^{\alpha-1}(1-\mu)^{\beta-1}$

$p(x|\mu)\cdot p(\mu|\alpha,\beta) =$

$= \mu^{x}(1-\mu)^{1-x}\cdot \mu^{\alpha-1}(1-\mu)^{\beta-1} =$

$= \mu^{x+\alpha-1}(1-\mu)^{\beta-x} = p(\mu|x+\alpha, \beta-x+1)$

$$p(X|\mu) = \prod_{i=1}^{N} \mu^{x_i}(1-\mu)^{1-x_i}$$

$p(\mu|X) \propto \underbrace{p(\mu)}_{\text{Beta}}\, p(X|\mu) =$

$= p(\mu|\alpha+\sum x_i, \beta+N-\sum x_i)$

Верно

$x_0 \sim N(\mu_0, \Sigma_0)$ $\quad x_t = A x_{t-1} + w$, $w \sim N(0, Q)$

$y_t = C x_t + v$, $v \sim N(0, R)$

$\boxed{6.5}$ a) Gausian form for $p(x_1, x_2, \ldots, x_T)$

$\because$ Gaus is linear

$p(x_0) = N_0(\mu_0, \Sigma_0)$

$p(x_1) =$

$x_1 = A x_0 + w \sim N$

$x_0 \sim N_0$ $\quad w \sim N(0, Q)$

$x_2 \sim N$

...

b) $\exists\, p(x_t|y_1 \ldots y_t) = N(\mu_t, \Sigma_t)$

1) $p(x_{t+1}|\vec{y})$ - ?

$p(A x_t + w | y_1, \ldots y_t) =$

$=$?

$x_2 = A x_1 + w =$ ($\sim N$)

$= A(A x_0 + w) + w$

$= A^2 x_0 + (A+1) w$

$x_3 = A x_2 + w =$ ($\sim N$)

$= A^3 x_0 + (A^2 + A + 1) w$

$\sim N$ $\quad \sim N$

a) Form of $p(x_0, x_1, \ldots, x_T)$ is Gaussian $\because$ joint is N.

$\sim N$ $\quad \sim N$ $\quad \sim N$

~~Верно~~

b) $\exists\, p(x_t|y_1, \ldots, y_t) = N(\mu_t, \Sigma_t)$

1. $p(x_{t+1}|y_1 \ldots, y_t)$ - ?

$\cancel{p(x_{t+1}|\vec{y}) = \frac{p(x_{t+1}, \vec{y})}{p(\vec{y})} = \frac{p(A x_t + w, \vec{y})}{p(\vec{y})}}$

$\cancel{= \frac{p(A x_t, \vec{y}) + p(w, \vec{y})}{p(\vec{y})}}$?

b

$\exists\, p(\vec{x}_t|\vec{y}_1, \ldots, \vec{y}_t) = N(\mu_t, \Sigma_t)$

$p(\vec{x}_{t+1}|\vec{y}_{1:t}) = N(\vec{x}_{t+1} | \mu,$

$\vec{x}_{t+1} = A x_t + w$