$\boxed{7.1}$

$f(x) = x^3 + 6x^2 - 3x - 5$

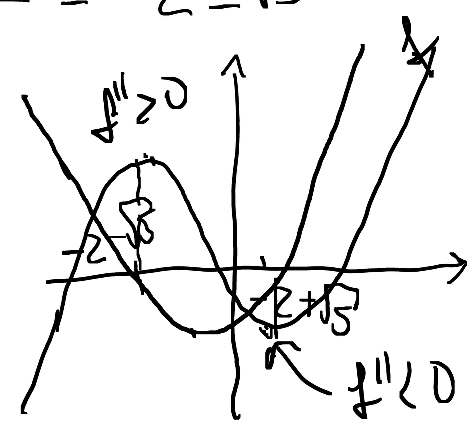
$f' = \cancel{3}x^2 + \cancel{12}^{4}x - \cancel{3}^{1} = 0$

$x_{1,2} = \frac{-4 \pm \sqrt{16+4}}{2} = -2 \pm \sqrt{5}$

$f(-2+\sqrt{5}) \approx -5.36$

$f(-2-\sqrt{5}) \approx 39$

$f'' = \cancel{2}^{6}x + \cancel{4}^{12} = 0, \; x = -2$



$f''(-2+\sqrt{5}) = \cancel{-4} + 2\sqrt{5} \cancel{+4} > 0$

$f''(-2-\sqrt{5}) < 0$

Min: $-2+\sqrt{5}$

Max: $-2-\sqrt{5}$

Верно

$\boxed{7.2}$

$\theta_{i+1} = \theta_i - \gamma_i (\nabla L(\theta_i))^T =$

$= \theta_i - \gamma_i \sum_{n=1}^{N} (\nabla L_n(\theta_i))^T$

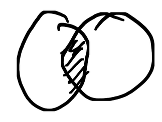
$\nabla L_n(\theta_i) = \nabla(-\Sigma \log p(y_n | x_n, \theta_i))$?
regression

For $N=1$, $\theta_{i+1} = \theta_i - \gamma_i \left(\frac{d L_n(\theta_i)}{d \theta_i} \in \mathbb{R}^{1\times D}\right)^T$

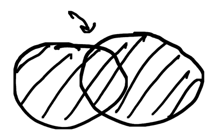
Верно

7.3

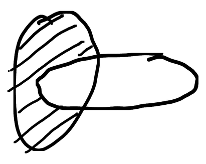
a) Yes:



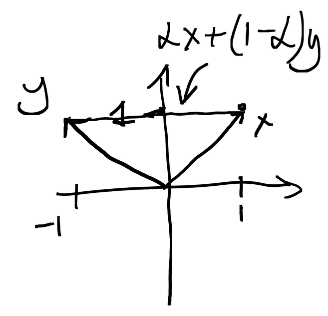
b) No:



c) No:



верно



$x=1,\ y=100$

	αx	$(1-\alpha)y$	$\alpha x+(1-\alpha)y$
$\alpha=0$	0	y	y
$\alpha=1/2$	$\frac{x}{2}$	$\frac{y}{2}$	$\frac{x+y}{2}$
$\alpha=1/3$	$\frac{x}{3}$	$\frac{2y}{3}$	$\frac{x+2y}{3}$
$\alpha=1$	x	0	x

$\to f(x)=x$

$f(\alpha x+(1-\alpha)y)=$

$=\alpha x+(1-\alpha)y=$

$=\alpha f(x)+(1-\alpha)f(y)$

$\to f(x)=x^2$ ✓

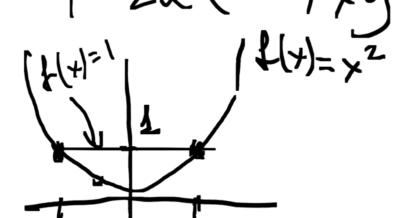
$(\alpha x+(1-\alpha)y)^2=$

$=\alpha^2x^2+2\alpha(1-\alpha)xy+$

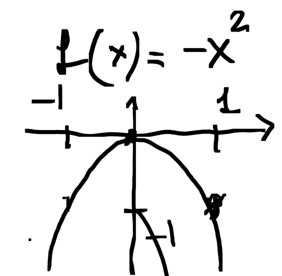
$+(1-\alpha)^2y^2=$

$=\alpha^2 f(x)+(1-\alpha)^2 f(y)+$

$+\ 2\alpha(1-\alpha)xy$



$\to f(x)=-x^2$



$\alpha=\frac{1}{2}$

$0=f(-\frac{1}{2}+\frac{1}{2})\nleq$

$\frac{1}{2}f(-1)+\frac{1}{2}f(1)=$

$=-1$

7.4

a) $f(x)+g(x)$ - convex? $h(x)=f(x)+g(x)$

$h(\alpha x+(1-\alpha)y)=$

$=f(\alpha x+(1-\alpha)y)+g(\alpha x+(1-\alpha)y)\leqslant$

$\leqslant\alpha f(x)+(1-\alpha)f(y)+\alpha g(x)+(1-\alpha)g(y)$

$=\alpha(f(x)+g(x))+(1-\alpha)(f(y)+g(y))$

$=\alpha h(x)+(1-\alpha)h(y)$ ✓ Yes

b) $f(x)-g(x)$ - convex?

$f-g-(\alpha f(x)+(1-\alpha)f(y)-$

$-(\alpha g(x)+(1-\alpha)g(y)))=$

$=f-g-\alpha(f(x)-g(x))-$

$-(1-\alpha)(f(y)-g(x)))$

$f'=f-\alpha f(x)-(1-\alpha)f(y)\leqslant 0$

$g'=(g-\alpha g(x)-(1-\alpha)g(y))\leqslant 0$

$\exists\ f'>g'$

$f'-g'\nleq 0\Rightarrow$ No

c) $f(x)\cdot g(x)$ - convex?

$f\cdot g-(\alpha^2 f(x)g(x)+\alpha(1-\alpha)f(x)g(y)+$

$+\alpha(1-\alpha)f(y)g(x)+(1-\alpha)^2 f(y)g(y))$

$\exists\ 0<f<1,\ 0<g<1$

$\boxed{7.5}$ $\max\limits_{x\in\mathbb{R}^2, \xi\in\mathbb{R}} \vec{p}^T\vec{x} + \xi$, $\xi \geq 0$, $x_0 \leq 0$, $x_1 \leq 3$

$\Downarrow$

$-\xi \leq 0$

$$\max \begin{bmatrix} p_1 \\ p_2 \\ 1 \end{bmatrix}^T \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \xi \end{bmatrix}},$$

$$\underbrace{\overbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}}^{3\times 3}}_{A} \begin{bmatrix} x_1 \\ x_2 \\ \xi \end{bmatrix} \leq \underbrace{\begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}}_{b}$$

верно

$\boxed{7.6}$

$$\min_{v \in \mathbb{R}^2} \underbrace{\begin{bmatrix} -5 \\ -3 \end{bmatrix}}_{c}^T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 2 & 2 \\ 2 & -4 \\ -2 & 1 \\ 0 & -1 \\ 0 & 1 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \leq \underbrace{\begin{bmatrix} 33 \\ 8 \\ 5 \\ -1 \\ 8 \end{bmatrix}}_{b}$$

na